Vector Algebra

1.

Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$.

Then, $\hat{a} \cdot \hat{b}$ is equal to :

(2024)

(A) $\pm \frac{3}{5}$

(B) $\pm \frac{3}{4}$

(C) $\pm \frac{4}{5}$

(D) $\pm \frac{4}{3}$

Ans.

(C) $\pm \frac{4}{5}$

2. The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(2024)

(A) $\hat{i} - \hat{j} + 2\hat{k}$

(B) $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$

(D) $-\hat{i} + \hat{j} - 2\hat{k}$

Ans.

(D) $-\hat{i} + \hat{j} - 2\hat{k}$

3.

If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\,|\vec{a}|$.

(2024)

Ans.

$(\vec{a} + \vec{b}).\vec{a} = 0 \Rightarrow |\vec{a}|^2 + \vec{b}.\vec{a} = 0$ -----------------(1)

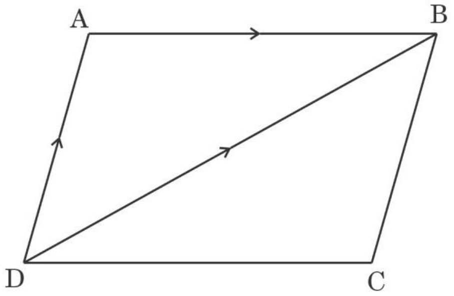
$(2\vec{a} + \vec{b}).\vec{b} = 0 \Rightarrow 2\vec{a}.\vec{b} + |\vec{b}|^2 = 0$ -------------(2)

$2(-|\vec{a}|^2) + |\vec{b}|^2 = 0$ {Using (1) and (2)}

$|\vec{b}|^2 = 2|\vec{a}|^2 \Rightarrow |\vec{b}| = \sqrt{2}|\vec{a}|$

4.

In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.



(2024)

Ans.

$$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$$

$$\overrightarrow{AD} = (2\hat{i} - 4\hat{j} + 5\hat{k}) - (3\hat{i} - 6\hat{j} + 2\hat{k})$$

$$= -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{AD} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & -4 & 5 \end{vmatrix} = 22\hat{i} + 11\hat{j}$$

$$\text{Area} = |\overrightarrow{AD} \times \overrightarrow{AB}| = |22\hat{i} + 11\hat{j}|$$

$$= \sqrt{605} \text{ or } 11\sqrt{5}$$

Previous Years' CBSE Board Questions

10.2 Some Basic Concepts

VSA (1 mark)

1. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis. *(AI 2014)* Ev

SA I (2 marks)

2. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units. *(2020)* An

10.3 Types of Vectors

MCQ

3. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if
   (a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$
   (c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$
   (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$ *(2023)*

4. The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is
   (a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\sqrt{3}$ *(2020)* Ap

10.4 Addition of Vectors

MCQ

5. $ABCD$ is a rhombus, whose diagonals intersect at E. Then $\overline{EA} + \overline{EB} + \overline{EC} + \overline{ED}$ equals
   (a) $\vec{0}$ (b) $\overline{AD}$ (c) $2\overline{BC}$ (d) $2\overline{AD}$ *(2020)* Ap

10.5 Multiplication of a Vector by a Scalar

MCQ

6. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is
   (a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$ (b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$
   (c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$ (d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$ *(2023)*

VSA (1 mark)

7. The position vector of two points A and B are $\overline{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overline{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is ______. *(2020)* An

8. Find the position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2 : 1. *(Delhi 2016)* An

9. Write the position vector of the point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $2\vec{a} + 3\vec{b}$ in the ratio 2 : 1. *(AI 2016)*

10. Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$. *(Foreign 2015)*

11. Find a vector in the direction of $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units. *(Delhi 2015C)*

12. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$ where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$. *(AI 2015C)* An

13. Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$. *(Delhi 2014)*

14. Find the value of 'p' for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel. *(AI 2014)* An

15. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units. *(Foreign 2014)*

16. Write a unit vector in the direction of vector $\overline{PQ}$, where P and Q are the points(1, 3, 0) and (4, 5, 6) respectively. *(Foreign 2014)*

17. Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units. *(Delhi 2014C)* An

SA I (2 marks)

18. X and Y are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally. *(AI 2019)* Cr

LA I (4 marks)

19. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A. *(Delhi 2016, Foreign 2015)*

10.6 Product of Two Vectors

MCQ

20. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when
    (a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$
    (c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$ *(2023)*

21. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is
    (a) 1 (b) 5 (c) 7 (d) 12 *(2023)*

22. If the projection of $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ on $\vec{b}=2\hat{i}+\lambda\hat{k}$ is zero, then the value of λ is
(a) 0 (b) 1
(c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$ *(2020)* An

23. If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then
(a) $\hat{i}\cdot\hat{j}=1$ (b) $\hat{i}\times\hat{j}=1$
(c) $\hat{i}\cdot\hat{k}=0$ (d) $\hat{i}\times\hat{k}=0$ *(2020)* Ap

VSA (1 mark)

24. Find the magnitude of vector $\vec{a}$ given by $\vec{a}=(\hat{i}+3\hat{j}-2\hat{k})\times(-\hat{i}+3\hat{k})$. *(2021C)*

25. If $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=5\hat{i}-3\hat{j}-4\hat{k}$ then find the ratio $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$. *(2020C)*

26. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is ________ square units. *(2020)*

27. The value of λ for which the vectors $2\hat{i}-\lambda\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$ are orthogonal is ________. *(2020)*

28. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$. *(2018)* An

29. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. *(AI 2016)*

30. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then write the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. *(NCERT, Foreign 2016)*

31. If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=400$ and $|\vec{a}|=5$ then write the value of $|\vec{b}|$. *(Foreign 2016)*

32. If $\vec{a}=7\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$, then find the projection of $\vec{a}$ on $\vec{b}$. *(Delhi 2015)*

33. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a}+\hat{b}+\hat{c}|$. *(AI 2015)* Ev

34. Write a unit vector perpendicular to both the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. *(AI 2015)*

35. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i}-3\hat{k}$ and $4\hat{j}+2\hat{k}$. *(Foreign 2015)*

36. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\sqrt{2}\vec{a}-\vec{b}$ is a unit vector? *(Delhi 2015C)* An

37. Find the projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$. *(AI 2015C)*

38. Find the projection of vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}-3\hat{j}+6\hat{k}$. *(Delhi 2014)* U

39. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. *(Delhi 2014)* Ap

40. If vectors $\vec{a}$ and $\vec{b}$ are such that, $|\vec{a}|=3, |\vec{b}|=\frac{2}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector, then write the angle between $\vec{a}$ and $\vec{b}$. *(Delhi 2014)* Cr

41. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5$, find the value of $|\vec{b}|$. *(AI 2014)* An

42. Write the projection of the vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$. *(Foreign 2014)*

43. Write the value of $\hat{i}\times(\hat{j}+\hat{k})+\hat{j}\times(\hat{k}+\hat{i})+\hat{k}\times(\hat{i}+\hat{j})$. *(Foreign 2014)*

44. Write the projection of the vector $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$. *(Delhi 2014C)* Ap

45. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$, given that $(\sqrt{3}\vec{a}-\vec{b})$ is a unit vector. *(Delhi 2014C)*

46. Write the value of cosine of the angle which the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ makes with *y*-axis. *(Delhi 2014C)* Ap

47. If $|\vec{a}|=8, |\vec{b}|=3$ and $|\vec{a}\times\vec{b}|=12$, find the angle between $\vec{a}$ and $\vec{b}$. *(AI 2014C)*

48. Find the angle between *x*-axis and the vector $\hat{i}+\hat{j}+\hat{k}$. *(AI 2014C)* Cr

SA I (2 marks)

49. If $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$, then find a unit vector along the vector $\vec{a}\times\vec{b}$. *(2023)*

50. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=3, |\vec{b}|=\frac{2}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. *(2023)*

51. Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$. *(2023)*

52. Write the projection of the vector $(\vec{b}+\vec{c})$ on the vector $\vec{a}$ where, $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. *(Term II, 2021-22)* Ap

53. If $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-\hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a}+\vec{b})$ and $(\vec{b}-\vec{c})$. *(Term II, 2021-22C)*

54. $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a}+3\vec{b}| = |3\vec{a}-2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$. *(Term II, 2021-22)*

55. If $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 400$ and $|\vec{b}|=5$, then find the value of $|\vec{a}|$. *(Term II, 2021-22)* Ap

56. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then find $|\vec{b}|$. *(Term II, 2021-22)*

57. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. *(Term II, 2021-22C)*

58. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}|=|\vec{b}|$, then prove that $(\vec{a}+2\vec{b})$ is perpendicular to $\vec{a}$. *(Term II, 2021-22)*

59. If the sides AB and BC of a parallelogram $ABCD$ are represented as vectors $\overline{AB}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\overline{BC}=\hat{i}+2\hat{j}+3\hat{k}$, then find the unit vector along diagonal AC. *(2021C)*

60. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{b} = 7\hat{i}+6\hat{j}+2\hat{k}$. *(2020)*

61. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$. *(2020 C)*

62. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors. *(2020)*

63. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, 3\hat{i}+7\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+2\hat{k}$ form the sides of a right-angled triangle. *(2020)*

64. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$. *(Delhi 2019)*

65. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ be two vectors. Show that the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other. *(AI 2019)*

66. Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ where $\vec{a} = 4\hat{i}-\hat{j}+8\hat{k}$ and $\vec{b} = -\hat{j}+\hat{k}$ *(2019 C)*

67. If $|\vec{a}|=2, |\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$. *(2019)* Ap

68. For any two vectors, $\vec{a}$ and $\vec{b}$, prove that $(\vec{a}\times\vec{b})^2 = \vec{a}^2\vec{b}^2-(\vec{a}\cdot\vec{b})^2$ *(2019)* An

69. If θ is the angle between two vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$, find $\sin\theta$. *(2018)*

SA II (3 marks)

70. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$, then find a unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. *(2023)*

71. Three vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=2$. *(2023)*

72. The two adjacent sides of a parallelogram are represented by $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also. *(Term II, 2021-22)*

73. If $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude, then prove that the vector $(2\vec{a}+\vec{b}+2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$. Also, find the angle between $\vec{a}$ and $(2\vec{a}+\vec{b}+2\vec{c})$. *(Term II, 2021-22)*

74. If $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=4$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$ *(Term II, 2021-22)*

75. If $\vec{a}$ and $\vec{b}$ are two vectors of equal magnitude and α is the angle between them, then prove that $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}=\cot\left(\frac{\alpha}{2}\right)$ *(Term II, 2021-22)*

LA I (4 marks)

76. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. *(2020)*

77. Using vectors, find the area of the triangle ABC with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$ *(NCERT Exemplar, 2020)*

78. Prove that three points A, B and C with position vectors $\vec{a},\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})+(\vec{a}\times\vec{b})=\vec{0}$. *(2020 C)*

79. The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$ *(2019, AI 2014)*

80. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overline{AB}$ and $\overline{CD}$ are collinear or not. *(Delhi 2019)*

81. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=1, |\vec{b}|=2, |\vec{c}|=3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$; and $\vec{b},\vec{c}$ are perpendicular to each other, then find $|3\vec{a}-2\vec{b}+2\vec{c}|$. *(2019)*

82. Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}, \vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a}=21$. *(2018)*

83. If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$. *(Delhi 2017)*

84. Show that the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively,are the vertices of a right-angled triangle. Hence find the area of the triangle. *(AI 2017)*

85. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram. *(AI 2016)*

86. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$. *(Foreign 2016)*

87. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, find $(\vec{r}\times\hat{i})\cdot(\vec{r}\times\hat{j})+xy$. *(Delhi 2015)*

88. If $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{i}-4\hat{j}-5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a}-\vec{b})$ and $(\vec{c}-\vec{b})$. *(AI 2015)*

89. Vectors $\vec{a},\vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$. *(Delhi 2014)*

90. Find a unit vector perpendicular to both of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$. *(Foreign 2014)*

91. If $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=2\hat{j}-\hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a}+\vec{b})$ and $(\vec{b}+\vec{c})$. *(Delhi 2014C)*

92. Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha}=4\hat{i}+5\hat{j}-\hat{k}$ and $\vec{\beta}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{p}\cdot\vec{q}=21$, where $\vec{q}=3\hat{i}+\hat{j}-\hat{k}$. *(AI 2014C)*

CBSE Sample Questions

10.5 Multiplication of a Vector by a Scalar

VSA (1 mark)

1. Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$. *(2020-21)*

2. Vector of magnitude 5 units and in the direction opposite to $2\hat{i}+3\hat{j}-6\hat{k}$ is_____. *(2020-21)* U

10.6 Product of Two Vectors

MCQ

3. The scalar projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is
   (a) $\frac{7}{\sqrt{14}}$ (b) $\frac{7}{14}$ (c) $\frac{6}{13}$ (d) $\frac{7}{2}$ *(2022-23)*

4. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=4$, then $|\vec{a}-2\vec{b}|$ is equal to
   (a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$ *(2022-23)*

VSA (1 mark)

5. Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$. *(2020-21)* Ap

6. Find the angle between the unit vectors $\hat{a}$ and $\hat{b}$, given that $|\hat{a}+\hat{b}|=1$. *(2020-21)*

SA I (2 marks)

7. Find $|\vec{x}|$, if $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$, where $\vec{a}$ is a unit vector. *(2022-23)* An

8. If $\vec{a}$ and $\vec{b}$ are unit vectors, then prove that $|\vec{a}+\vec{b}|=2\cos\frac{\theta}{2}$, where θ is the angle between them. *(Term II, 2021-22)* Ev

9. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i}-\hat{j}+\hat{k}$ and $4\hat{i}+5\hat{k}$ respectively. *(2020-21)*

SA II (3 marks)

10. If $\vec{a}\neq\vec{0}$, $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, then show that $\vec{b}=\vec{c}$. *(Term II, 2021-22)* An

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. Here, $l=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$, $m=\cos\frac{\pi}{2}=0$, $n=\cos\theta$

Since, $l^2+m^2+n^2=1$

$\Rightarrow \frac{1}{2}+0+\cos^2\theta=1 \Rightarrow \cos^2\theta=1-\frac{1}{2}=\frac{1}{2}$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$

Commonly Made Mistake

- $\cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4} \quad \therefore \theta = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore$ The vector of magnitude $5\sqrt{2}$ is

$\vec{a} = 5\sqrt{2}(l\hat{i} + m\hat{j} + n\hat{k})$

$= 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right) = 5(\hat{i} + \hat{k})$ [$\because \theta$ is an acute angle]

2. We have, $|\vec{r}| = 3\sqrt{3}$

Since, $\vec{r}$ is equally inclined to three axes, so direction cosine of unit vector $\vec{r}$ will be same. *i.e.*, $l = m = n$

As we know that $l^2 + m^2 + n^2 = 1$ ...(i)

$l^2 + l^2 + l^2 = 1 \Rightarrow 3l^2 = 1$

$\Rightarrow l = \pm\frac{1}{\sqrt{3}} = m = n$ ...(ii)

We have $\overline{OP} = \pm\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$ $\left\{\because \hat{p} = \frac{\vec{r}}{|\vec{r}|}\right\}$

$\vec{r} = |\vec{r}|\overline{OP}$ $\{\because |\vec{r}| = 3\sqrt{3} \text{ (given)}\}$

$= \pm 3\sqrt{3} \times \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) \Rightarrow \vec{r} = \pm 3(\hat{i} + \hat{j} + \hat{k})$

Answer Tips

- If a vector is equally inclined to axes, then its direction cosines are equal.

3. (b)

4. (b) : Let $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$

So, unit vector of $\vec{a} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

$\therefore$ The value of p is $\frac{1}{\sqrt{3}}$.

5. (a) : $\overline{EA} + \overline{EB} + \overline{EC} + \overline{ED} = \overline{EA} + \overline{EB} - \overline{EA} - \overline{EB}$

[As diagonals of a rhombus bisect each other]

$= \vec{0}$

6. (b): Let $\vec{v} = 4\hat{i} - 3\hat{k}$

$\therefore |\vec{v}| = \sqrt{4^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Now, $\hat{v}$ = unit vector along $\vec{v}$

$= \frac{\vec{v}}{|\vec{v}|} = \frac{1}{5}(4\hat{i} - 3\hat{k})$

7. Required position vector of point P

$= \frac{1(2\hat{i} - \hat{j} - \hat{k}) + 2(2\hat{i} - \hat{j} + 2\hat{k})}{2+1} = \frac{2\hat{i} - \hat{j} - \hat{k} + 4\hat{i} - 2\hat{j} + 4\hat{k}}{3}$

$= \frac{1}{3}(6\hat{i} - 3\hat{j} + 3\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}$

Concept Applied

- If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B respectively, then the position vector of $R(\vec{r})$ which divides $\overline{AB}$ internally in the ratio $m : n$ is $\frac{m\vec{b} + n\vec{a}}{m+n}$

8. Required position vector

$= \frac{2\cdot(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})}{2-1} = \frac{4\vec{a} + 2\vec{b} - \vec{a} + 2\vec{b}}{1} = 3\vec{a} + 4\vec{b}$

9. Required position vector

$= \frac{2(2\vec{a} + 3\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1} = \frac{7\vec{a} + 4\vec{b}}{3} = \frac{7}{3}\vec{a} + \frac{4}{3}\vec{b}$

10. Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = 4\hat{i} - 3\hat{j} + 2\hat{k}$.

Then, the sum of the given vectors is

$\vec{c} = \vec{a} + \vec{b} = (2+4)\hat{i} + (3-3)\hat{j} + (-1+2)\hat{k} = 6\hat{i} + \hat{k}$

and $|\vec{c}| = |\vec{a} + \vec{b}| = \sqrt{6^2 + 1^2} = \sqrt{36+1} = \sqrt{37}$

$\therefore$ Unit vector, $\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{6\hat{i} + \hat{k}}{\sqrt{37}} = \frac{6}{\sqrt{37}}\hat{i} + \frac{1}{\sqrt{37}}\hat{k}$

11. A unit vector in the direction of $\vec{a} = \hat{i} - 2\hat{j}$ is $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

$= \frac{\hat{i} - 2\hat{j}}{\sqrt{1^2 + (-2)^2}} = \frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$

$\therefore$ The required vector of magnitude 7 in the direction of $\vec{a} = 7\cdot\hat{a} = \frac{7}{\sqrt{5}}(\hat{i} - 2\hat{j})$.

12. $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$; $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\therefore 3\vec{a} + 2\vec{b} = 3(\hat{i} + \hat{j} - 2\hat{k}) + 2(2\hat{i} - 4\hat{j} + 5\hat{k})$

$= (3\hat{i} + 3\hat{j} - 6\hat{k}) + (4\hat{i} - 8\hat{j} + 10\hat{k}) = 7\hat{i} - 5\hat{j} + 4\hat{k}$

$\therefore$ The direction ratios of the vector $3\vec{a} + 2\vec{b}$ are 7, –5, 4.

13. We have, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

Sum of the given vectors is

$\vec{c} = \vec{a} + \vec{b} = (2+2)\hat{i} + (2+1)\hat{j} + (-5-7)\hat{k} = 4\hat{i} + 3\hat{j} - 12\hat{k}$

and $|\vec{c}| = \sqrt{(4)^2 + (3)^2 + (-12)^2} = \sqrt{169} = 13$

$\therefore$ Unit vector, $\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{13} = \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$

14. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} - 2p\hat{j} + 3\hat{k}$

For $\vec{a}$ and $\vec{b}$ to be parallel, $\vec{b} = \lambda\vec{a}$.

$\Rightarrow \hat{i} - 2p\hat{j} + 3\hat{k} = \lambda(3\hat{i} + 2\hat{j} + 9\hat{k}) = 3\lambda\hat{i} + 2\lambda\hat{j} + 9\lambda\hat{k}$

$\Rightarrow 1 = 3\lambda, -2p = 2\lambda, 3 = 9\lambda \Rightarrow \lambda = \frac{1}{3}$ and $p = -\lambda = -\frac{1}{3}$

Concept Applied

- Two vectors $\vec{a}$ and $\vec{b}$ are parallel iff one of them is scalar multiple of other.

15. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

A vector in the direction of $\vec{a}$ with a magnitude of $21 = 21\,\hat{a}$

$\therefore$ Required vector $= 21 \times \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{2^2 + (-3)^2 + 6^2}}$

$= 21 \times \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7} = 6\hat{i} - 9\hat{j} + 18\hat{k}$

16. We have, $\overline{PQ} = \overline{OQ} - \overline{OP}$

$= (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 3\hat{j}) = 3\hat{i} + 2\hat{j} + 6\hat{k}$

Required unit vector $= \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + 2^2 + 6^2}} = \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{7}$

Concept Applied

- Unit vector of $\overline{PQ} = \dfrac{\overline{PQ}}{|\overline{PQ}|}$

17. Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

The vector in the direction of $\vec{a}$ with magnitude 9 units $= 9\hat{a}$

$\therefore$ Required vector $= 9 \times \dfrac{\vec{a}}{|\vec{a}|} = 9 \times \dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}}$

$= \dfrac{9}{3}(\hat{i} - 2\hat{j} + 2\hat{k}) = 3\hat{i} - 6\hat{j} + 6\hat{k}$

Answer Tips

- The vector in direction of $\vec{a} = |\vec{a}| \cdot \hat{a}$

18. Position vector which divides the line segment joining points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ in the ratio 2 : 1 externally is given by

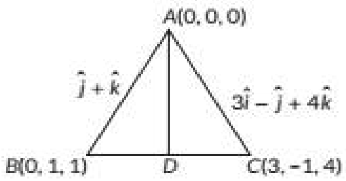
$\dfrac{2(\vec{a} - 3\vec{b}) - 1(3\vec{a} + \vec{b})}{2-1} = \dfrac{2\vec{a} - 6\vec{b} - 3\vec{a} - \vec{b}}{1} = -\vec{a} - 7\vec{b}$

Concept Applied

- If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B respectively, then the position vector of $R(\vec{r})$ which divides $\overline{AB}$ externally in the ratio $m : n$ is $\dfrac{m\vec{b} - n\vec{a}}{m-n}$.

19. Take A to be as origin (0, 0, 0).

$\therefore$ Coordinates of B are (0, 1, 1) and coordinates of C are (3, –1, 4).



Let D be the mid point of BC and AD is a median of ΔABC.

$\therefore$ Coordinates of D are $\left(\dfrac{3}{2}, 0, \dfrac{5}{2}\right)$

So, length of $AD = \sqrt{\left(\dfrac{3}{2} - 0\right)^2 + (0)^2 + \left(\dfrac{5}{2} - 0\right)^2}$

$= \sqrt{\dfrac{9}{4} + \dfrac{25}{4}} = \dfrac{\sqrt{34}}{2}$ units

Concept Applied

- Position vector of mid point of $AB = \dfrac{\vec{a} + \vec{b}}{2}$

20. (b): Given, $\vec{a} \cdot \vec{b} \geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

Assuming $|\vec{a}| \neq 0$ and $|\vec{b}| \neq 0$

$\Rightarrow \cos\theta \geq 0$ $[\because |\vec{a}| \geq 0, |\vec{b}| \geq 0]$ $\Rightarrow \theta \in \left[0, \dfrac{\pi}{2}\right]$

21. (c): Given vector is $6\hat{i} - 2\hat{j} + 3\hat{k}$

$\therefore$ Its magnitude $= \sqrt{6^2 + (-2)^2 + 3^2}$

$= \sqrt{36 + 4 + 9} = \sqrt{49} = 7$ units

22. (c): Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \lambda\hat{k}$

Since, projection of $\vec{a}$ on $\vec{b} = 0$

$\Rightarrow \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 0 \Rightarrow \dfrac{(\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + \lambda\hat{k})}{\sqrt{2^2 + \lambda^2}} = 0$

$\Rightarrow \dfrac{2 + 3\lambda}{\sqrt{4 + \lambda^2}} = 0 \Rightarrow 2 + 3\lambda = 0 \Rightarrow \lambda = -\dfrac{2}{3}$

23. (c): Since, $\hat{i}, \hat{j}, \hat{k}$ are mutually perpendicular to each other.

$\therefore \hat{i} \cdot \hat{k} = 0$

24. Let $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 0\hat{j} + 3\hat{k})$

$\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix} = (9-0)\hat{i} - (3-2)\hat{j} + (0+3)\hat{k} = 9\hat{i} - \hat{j} + 3\hat{k}$

$\therefore |\vec{a}| = \sqrt{9^2 + (-1)^2 + 3^2} = \sqrt{91}$

Answer Tips

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

25. Here $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$

Since projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

and projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

$\therefore \vec{a} \cdot \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) \cdot (5\hat{i} - 3\hat{j} - 4\hat{k})$

$\vec{a} \cdot \vec{b} = 10 + 3 - 8 = 13 - 8 = 5$

$|\vec{a}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$

$|\vec{b}| = \sqrt{5^2 + (-3)^2 + (-4)^2} = \sqrt{25 + 9 + 16} = \sqrt{50} = 5\sqrt{2}$

Required ratio $= \dfrac{5/5\sqrt{2}}{5/3} = \dfrac{5}{5\sqrt{2}} \times \dfrac{3}{5} = \dfrac{3\sqrt{2}}{10}$

26. Given, two diagonals $\vec{d}_1$ and $\vec{d}_2$ are $2\hat{i}$ and $-3\hat{k}$ respectively.

$\therefore \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 0 & -3 \end{vmatrix} = \hat{i}(0) - \hat{j}(-6-0) + \hat{k}(0) = 6\hat{j}$

So, area of the parallelogram = $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$= \frac{1}{2} \times 6 = 3$ sq. units

27. Let $\vec{a} = 2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

We know, $\vec{a}$ and $\vec{b}$ are orthogonal iff $\vec{a} \cdot \vec{b} = 0$

$\Rightarrow (2\hat{i} - \lambda\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 0$

$\Rightarrow 2 - 2\lambda - 1 = 0 \Rightarrow 1 - 2\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$

28. Given, $|\vec{a}| = |\vec{b}|, \theta = 60°$ and $\vec{a} \cdot \vec{b} = \frac{9}{2}$

Now, $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos 60° = \frac{9/2}{|\vec{a}|^2} \Rightarrow \frac{1}{2} = \frac{9/2}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 9 \Rightarrow |\vec{a}| = 3 \therefore |\vec{a}| = |\vec{b}| = 3$

Answer Tips

- $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$

29. Given, $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$

Unit vectors perpendicular to $\vec{a}$ and $\vec{b}$ are $\pm\left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}\right)$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -\hat{i} - 2\hat{j} + 2\hat{k}$

$\therefore$ Unit vectors perpendicular to $\vec{a}$ and $\vec{b}$ are

$$\pm \frac{(-\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{(-1)^2 + (-2)^2 + (2)^2}} = \pm\left(-\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$$

So, there are two unit vectors perpendicular to the given vectors.

30. We have $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors.

Therefore, $|\vec{a}| = 1, |\vec{b}| = 1$ and $|\vec{c}| = 1$

Also, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ (given)

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -\frac{3}{2}$

31. $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400 \Rightarrow \{|\vec{a}||\vec{b}|\sin\theta\}^2 + \{|\vec{a}||\vec{b}|\cos\theta\}^2 = 400$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2 \sin^2\theta + |\vec{a}|^2|\vec{b}|^2 \cos^2\theta = 400$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2 = 400 \Rightarrow 25 \times |\vec{b}|^2 = 400$ $[\because |\vec{a}| = 5]$

$\Rightarrow |\vec{b}|^2 = 16 \Rightarrow |\vec{b}| = 4$

32. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{(7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = \frac{14 + 6 - 12}{7} = \frac{8}{7}$

33. Here $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors.

$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{c}| = 1$ and $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = 0$ ...(1)

$\therefore |2\hat{a} + \hat{b} + \hat{c}|^2 = (2\hat{a} + \hat{b} + \hat{c}) \cdot (2\hat{a} + \hat{b} + \hat{c})$

$= 4\hat{a} \cdot \hat{a} + 2\hat{a} \cdot \hat{b} + 2\hat{a} \cdot \hat{c} + 2\hat{b} \cdot \hat{a} + \hat{b} \cdot \hat{b} + \hat{b} \cdot \hat{c} + 2\hat{c} \cdot \hat{a} + \hat{c} \cdot \hat{b} + \hat{c} \cdot \hat{c}$

$= 4|\hat{a}|^2 + |\hat{b}|^2 + |\hat{c}|^2 + 4\hat{a} \cdot \hat{b} + 2\hat{b} \cdot \hat{c} + 4\hat{a} \cdot \hat{c}$

$(\because \hat{b} \cdot \hat{a} = \hat{a} \cdot \hat{b}, \hat{c} \cdot \hat{a} = \hat{a} \cdot \hat{c}, \hat{c} \cdot \hat{b} = \hat{b} \cdot \hat{c})$

$= 4 \cdot 1^2 + 1^2 + 1^2$ [Using (1)]

$= 6$

$\therefore |2\hat{a} + \hat{b} + \hat{c}| = \sqrt{6}.$

34. Here, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$

Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j} + 0\hat{k} = -\hat{i} + \hat{j}$

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$= \pm \frac{-\hat{i} + \hat{j}}{\sqrt{(-1)^2 + 1^2}} = \pm \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j}).$

Key Points

- Unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

35. Let $\vec{a} = 2\hat{i} - 3\hat{k}$ and $\vec{b} = 4\hat{j} + 2\hat{k}$

The area of a parallelogram with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by $|\vec{a} \times \vec{b}|$.

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{vmatrix} = 12\hat{i} - 4\hat{j} + 8\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(12)^2 + (-4)^2 + (8)^2} = \sqrt{144 + 16 + 64}$

$= \sqrt{224} = 4\sqrt{14}$ sq. units.

36. Let θ be the angle between the unit vectors $\vec{a}$ and $\vec{b}$.

$\therefore \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \vec{a} \cdot \vec{b}$ $(\because |\vec{a}| = 1 = |\vec{b}|)$ ...(i)

Now, $1 = |\sqrt{2}\vec{a} - \vec{b}|$

$\Rightarrow 1 = |\sqrt{2}\vec{a} - \vec{b}|^2 = (\sqrt{2}\vec{a} - \vec{b}) \cdot (\sqrt{2}\vec{a} - \vec{b})$

$= 2|\vec{a}|^2 - \sqrt{2}\vec{a} \cdot \vec{b} - \vec{b} \cdot \sqrt{2}\vec{a} + |\vec{b}|^2$

$= 2 - 2\sqrt{2}\vec{a} \cdot \vec{b} + 1$ $(\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a})$

$= 3 - 2\sqrt{2}\vec{a} \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ [By using (i)]

$\therefore \theta = \pi/4$

37. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{(2\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}+2\hat{j}+\hat{k})}{\sqrt{(2)^2+(2)^2+(1)^2}} = \frac{4+6+2}{\sqrt{9}} = \frac{12}{3} = 4$

38. Let $\vec{a} = \hat{i}+3\hat{j}+7\hat{k}, \vec{b} = 2\hat{i}-3\hat{j}+6\hat{k}$

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{(\hat{i}+3\hat{j}+7\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{(2)^2+(-3)^2+(6)^2}} = \frac{2-9+42}{\sqrt{49}} = \frac{35}{7} = 5$

39. Given $|\vec{a}|=1=|\vec{b}|, |\vec{a}+\vec{b}|=1$

$\Rightarrow |\vec{a}+\vec{b}|^2 = 1 \Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 1 \Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=1$

$\Rightarrow |\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2 = 1 \Rightarrow 1+2\vec{a}\cdot\vec{b}+1=1$

$\Rightarrow 2\vec{a}\cdot\vec{b} = -1 \Rightarrow 2|\vec{a}|\cdot|\vec{b}|\cos\theta = -1 \Rightarrow 2\cdot1\cdot1\cos\theta = -1$

$\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$

40. Given, $|\vec{a}|=3, |\vec{b}|=\frac{2}{3}$ and $|\vec{a}\times\vec{b}|=1$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 1 \Rightarrow 3.\frac{2}{3}\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

41. Given: $\vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b}=0$

Also, $|\vec{a}|=5$ and $|\vec{a}+\vec{b}|=13$

$\Rightarrow |\vec{a}+\vec{b}|^2 = 13^2 \Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 169$

$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b} = 169 \Rightarrow |\vec{a}|^2+0+0+|\vec{b}|^2 = 169$

$\Rightarrow |\vec{b}|^2 = 169-|\vec{a}|^2 = 169-5^2 = 144 \Rightarrow |\vec{b}| = 12$

42. The projection of the vector $(\hat{i}+\hat{j}+\hat{k})$ along the vector $\hat{j}$ is $(\hat{i}+\hat{j}+\hat{k})\cdot\left(\frac{\hat{j}}{\sqrt{0^2+1^2+0^2}}\right) = 1$

43. We have, $\hat{i}\times(\hat{j}+\hat{k})+\hat{j}\times(\hat{k}+\hat{i})+\hat{k}\times(\hat{i}+\hat{j})$

$= \hat{i}\times\hat{j}+\hat{i}\times\hat{k}+\hat{j}\times\hat{k}+\hat{j}\times\hat{i}+\hat{k}\times\hat{i}+\hat{k}\times\hat{j}$

$= \hat{k}-\hat{j}+\hat{i}-\hat{k}+\hat{j}-\hat{i} = \vec{0}.$

44. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{(2\hat{i}-\hat{j}+\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{(1)^2+(2)^2+(2)^2}} = \frac{2-2+2}{\sqrt{9}} = \frac{2}{3}$

45. Let θ be the angle between the unit vectors $\vec{a}$ and $\vec{b}$

$\therefore \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \vec{a}\cdot\vec{b}$ ...(i) $(\because |\vec{a}|=|\vec{b}|=1)$

Now, $|\sqrt{3}\vec{a}-\vec{b}| = 1 \Rightarrow |\sqrt{3}\vec{a}-\vec{b}|^2 = 1$

$\Rightarrow 3|\vec{a}|^2+|\vec{b}|^2-2\sqrt{3}\,\vec{a}\cdot\vec{b} = 1 \Rightarrow 3+1-2\sqrt{3}|\vec{a}||\vec{b}|\cos\theta = 1$

$\Rightarrow 3 = 2\sqrt{3}\cos\theta \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$

46. Let θ be the angle between the vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and y-axis *i.e.*, $\vec{b} = \hat{j}$

$\therefore \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$= \frac{(\hat{i}+\hat{j}+\hat{k}).\hat{j}}{|\hat{i}+\hat{j}+\hat{k}||\hat{j}|} = \frac{1}{\sqrt{1^2+1^2+1^2}\sqrt{1^2}} = \frac{1}{\sqrt{3}}$

47. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

Given: $|\vec{a}|=8, |\vec{b}|=3$ and $|\vec{a}\times\vec{b}|=12$

$\Rightarrow |\vec{a}||\vec{b}||\sin\theta| = 12 \Rightarrow 8\times3\times\sin\theta = 12$

$\Rightarrow \sin\theta = \frac{12}{24} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}.$

48. Here, $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and vector along x-axis is $\hat{i}$

$\therefore$ Angle between $\vec{a}$ and $\hat{i}$ is given by

$\cos\theta = \frac{\vec{a}.\hat{i}}{\sqrt{1^2+1^2+1^2}.\sqrt{1^2}} = \frac{(\hat{i}+\hat{j}+\hat{k})\cdot\hat{i}}{\sqrt{3}\cdot1} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

49. We have, $\vec{a} = 4\hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}-2\hat{j}+\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$$

$= \hat{i}(-1+2)-\hat{j}(4-2)+\hat{k}(-8+2) = \hat{i}-2\hat{j}-6\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{1^2+(-2)^2+(-6)^2} = \sqrt{41}$

Unit vector along $\vec{a}\times\vec{b} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$= \frac{\hat{i}-2\hat{j}-6\hat{k}}{\sqrt{41}} = \frac{1}{\sqrt{41}}\hat{i}-\frac{2}{\sqrt{41}}\hat{j}-\frac{6}{\sqrt{41}}\hat{k}$

50. We know that, $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow 1 = 3\times\frac{2}{3}\sin\theta$ $(\because \vec{a}\times\vec{b}$ is a unit vector$)$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin30^\circ \Rightarrow \theta = 30^\circ$

So, angle between $\vec{a}$ and $\vec{b}$ is 30°.

51. Area of parallelogram $= \vec{a}\times\vec{b} = \left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{matrix}\right\|$

$= |(-1+21)\hat{i}-(1-6)\hat{j}+(-7+2)\hat{k}| = |20\hat{i}+5\hat{j}-5\hat{k}|$

$= \sqrt{(20)^2+(5)^2+(-5)^2}$

$= \sqrt{400+25+25} = \sqrt{450} = 15\sqrt{2}$ sq.units

52.

$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$
$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$
$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

projection of $(\vec{b}+\vec{c})$ on $\vec{a}$ $= \dfrac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{a}|}$

$= \dfrac{(2\hat{i} - 2\hat{j} + \hat{k})\cdot(3\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{2^2 + (-2)^2 + 1^2}}$

$= \dfrac{(2\times3) + (-2\times1) + (1\times2)}{\sqrt{9}}$

$= \dfrac{6-2+2}{\sqrt{9}}$

$= \dfrac{6}{3}$

$= 2$ units

Answer: Projection of $(\vec{b}+\vec{c})$ on $\vec{a}$ = 2 units.

[Topper's Answer, 2022]

53. Here $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore\ \vec{a} + \vec{b} = (\hat{i} + \hat{j} - 2\hat{k}) + (-\hat{i} + 2\hat{j} + 2\hat{k}) = 3\hat{j}$

Now $(\vec{b} - \vec{c}) = (-\hat{i} + 2\hat{j} + 2\hat{k}) - (-\hat{i} + 2\hat{j} - \hat{k}) = 3\hat{k}$

Vector perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{b}-\vec{c})$ is

$$(\vec{a}+\vec{b}) \times (\vec{b}-\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \hat{i}(9-0) - \hat{j}(0-0) + \hat{k}(0-0) = 9\hat{i}$$

$\therefore$ Unit vector perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{b}-\vec{c})$

$$= \frac{9\hat{i}}{\sqrt{9^2+0^2+0^2}} = \frac{9\hat{i}}{9} = \hat{i} + 0\hat{j} + 0\hat{k}$$

54. Given $\vec{a}$ and $\vec{b}$ are unit vectors

$\therefore\ |\vec{a}| = |\vec{b}| = 1$ ...(i)

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Also, $|2\vec{a}+3\vec{b}| = |3\vec{a}-2\vec{b}|$ (Given)

$\Rightarrow |2\vec{a}+3\vec{b}|^2 = |3\vec{a}-2\vec{b}|^2$

$\Rightarrow (2\vec{a}+3\vec{b})\cdot(2\vec{a}+3\vec{b}) = (3\vec{a}-2\vec{b})\cdot(3\vec{a}-2\vec{b})$

$\Rightarrow 4(\vec{a}\cdot\vec{a}) + 6(\vec{a}\cdot\vec{b}) + 6(\vec{b}\cdot\vec{a}) + 9(\vec{b}\cdot\vec{b})$

$= 9(\vec{a}\cdot\vec{a}) - 6(\vec{a}\cdot\vec{b}) - 6(\vec{b}\cdot\vec{a}) + 4(\vec{b}\cdot\vec{b})$

$\Rightarrow 4|\vec{a}|^2 + 12(\vec{a}\cdot\vec{b}) + 9|\vec{b}|^2 = 9|\vec{a}|^2 - 12(\vec{a}\cdot\vec{b}) + 4|\vec{b}|^2$

$\Rightarrow 5|\vec{a}|^2 - 5|\vec{b}|^2 - 24|\vec{a}||\vec{b}|\cos\theta = 0$

$\Rightarrow 5\cdot1 - 5\cdot1 - 24\cos\theta = 0$ $(\because |\vec{a}| = |\vec{b}| = 1)$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

Answer Tips

- If $\vec{a}$ is a unit vector, then $|\vec{a}| = 1$

55. We have, $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 400$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2\sin^2\theta + |\vec{a}|^2|\vec{b}|^2\cos^2\theta = 400$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2 = 400$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow |\vec{a}|^2 \times 25 = 400 \Rightarrow |\vec{a}|^2 = \dfrac{400}{25} = 16 \Rightarrow |\vec{a}| = 4$

56. Given : $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a}\cdot\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{j} - \hat{k}$

To find $|\vec{b}|$.

Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Since, $\vec{a}\cdot\vec{b} = 1$

$\Rightarrow (\hat{i} + \hat{j} + \hat{k})\cdot(x\hat{i} + y\hat{j} + z\hat{k}) = 1 \Rightarrow x + y + z = 1$

and $\vec{a}\times\vec{b} = \hat{j} - \hat{k}$

$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \times (x\hat{i} + y\hat{j} + z\hat{k}) = \hat{j} - \hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k} \Rightarrow \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x) = \hat{j} - \hat{k}$$

$\Rightarrow x - z = 1, y - x = -1, z - y = 0$

$\Rightarrow z = y$ ...(1), $x - z = 1$ ...(2)

and $y - x = -1$ ...(3)

From equation (1), (2) and (3), we get

$x = 1, y = z = 0$

So $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k} = \hat{i}$ $|\vec{b}| = 1$.

57. Given, $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors.

$\therefore\ |\vec{a}| = 1 = |\vec{b}| = |\vec{c}|$

Also, given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c})\cdot(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \quad \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}\cdot\vec{c}=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(a.b+b.c+c.a)=0$

$\Rightarrow 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0 \Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=\frac{-3}{2}$

58. Given that $|\vec{a}+\vec{b}|=|\vec{b}|$

To prove : $(\vec{a}+2\vec{b})$ is perpendicular to $\vec{a}$.

i.e., $(\vec{a}+2\vec{b})\cdot\vec{a}=0$

Since, $|\vec{a}+\vec{b}|=|\vec{b}|$

Squaring both sides, we get $|\vec{a}+\vec{b}|^2=|\vec{b}|^2$

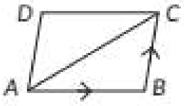
$\Rightarrow \quad (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=|\vec{b}|^2 \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{b}|^2$

$\Rightarrow \quad |\vec{a}|^2+2\vec{a}\cdot\vec{b}=0 \Rightarrow \vec{a}\cdot(\vec{a}+2\vec{b})=0 \Rightarrow (\vec{a}+2\vec{b})\cdot\vec{a}=0$

$\therefore \quad \vec{a}+2\vec{b}$ is perpendicular to $\vec{a}$.

59. Given, $\overline{AB}=2\hat{i}+4\hat{j}-5\hat{k}$, $\overline{BC}=\hat{i}+2\hat{j}+3\hat{k}$

Diagonal $\overline{AC}$ of parallelogram



$ABCD=\overline{AB}+\overline{BC}$

$\overline{AC}=(2\hat{i}+4\hat{j}-5\hat{k})+(\hat{i}+2\hat{j}+3\hat{k})=3\hat{i}+6\hat{j}-2\hat{k}$

Unit vector along diagonal $\overline{AC}=\frac{3\hat{i}+6\hat{j}-2\hat{k}}{|3\hat{i}+6\hat{j}-2\hat{k}|}$

$=\frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}}=\pm\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$

60. Here, $\vec{a}=5\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{b}=7\hat{i}+6\hat{j}+2\hat{k}$

Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 6 & -2\\ 7 & 6 & 2\end{vmatrix}$$

$=\hat{i}(12+12)-\hat{j}(10+14)+\hat{k}(30-42)$

$=24\hat{i}-24\hat{j}-12\hat{k}=12(2\hat{i}-2\hat{j}-\hat{k})$

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$=\pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\pm\frac{24\hat{i}-24\hat{j}-12\hat{k}}{\sqrt{576+576+144}}=\pm\frac{12(2\hat{i}-2\hat{j}-\hat{k})}{\sqrt{1296}}$

$=\pm\frac{12(2\hat{i}-2\hat{j}-\hat{k})}{36}=\pm\frac{1}{3}(2\hat{i}-2\hat{j}-\hat{k})$

61. Let $\vec{p}=|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ and $\vec{q}=|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$

Then we have $\vec{p}\cdot\vec{q}=\left[(|\vec{a}|\vec{b}+|\vec{b}|\vec{a})\cdot(|\vec{a}|\vec{b}-|\vec{b}|\vec{a})\right]$

$=|\vec{a}|^2(\vec{b}\cdot\vec{b})-|\vec{a}||\vec{b}|(\vec{b}\cdot\vec{a})+|\vec{b}||\vec{a}|(\vec{a}\cdot\vec{b})-|\vec{b}|^2(\vec{a}\cdot\vec{a})$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{a}||\vec{b}|(\vec{a}\cdot\vec{b})+|\vec{a}||\vec{b}|(\vec{a}\cdot\vec{b})-|\vec{b}|^2|\vec{a}|^2=0$ $(\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a})$

$\Rightarrow \quad \vec{p}\cdot\vec{q}=0$

Hence, $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$.

62. For any two non-zero vectors $\vec{a}$ and $\vec{b}$, we have

$|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\Leftrightarrow \quad |\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2 \Leftrightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$

$\Leftrightarrow \quad 4\vec{a}\cdot\vec{b}=0 \Leftrightarrow \vec{a}\cdot\vec{b}=0$

So, $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

63. Let $A(2\hat{i}-\hat{j}+\hat{k})$, $B(3\hat{i}+7\hat{j}+\hat{k})$ and $C(5\hat{i}+6\hat{j}+2\hat{k})$.

Then, $\overline{AB}=(3-2)\hat{i}+(7+1)\hat{j}+(1-1)\hat{k}=\hat{i}+8\hat{j}$

$\overline{AC}=(5-2)\hat{i}+(6+1)\hat{j}+(2-1)\hat{k}=3\hat{i}+7\hat{j}+\hat{k}$

$\overline{BC}=(5-3)\hat{i}+(6-7)\hat{j}+(2-1)\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

Now, angle between $\overline{AC}$ and $\overline{BC}$ is given by

$\Rightarrow \quad \cos\theta=\frac{\overline{AC}\cdot\overline{BC}}{|\overline{AC}||\overline{BC}|}=\frac{6-7+1}{\sqrt{9+49+1}\sqrt{4+1+1}}$

$\Rightarrow \quad \cos\theta=0 \Rightarrow \theta=90° \Rightarrow AC\perp BC$

So, *A*, *B*, *C* are the vertices of right angled triangle.

64. Given, $\hat{a}+\hat{b}=\hat{c}$

$\Rightarrow (\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})=\hat{c}\cdot\hat{c} \Rightarrow \hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b}=\hat{c}\cdot\hat{c}$

$\Rightarrow 1+\hat{a}\cdot\hat{b}+1+\hat{a}\cdot\hat{b}=1 \quad (\because \hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{a})$

$\Rightarrow 2\,\hat{a}\cdot\hat{b}=-1$ ...(i)

Now $|\hat{a}-\hat{b}|^2=(\hat{a}-\hat{b})\cdot(\hat{a}-\hat{b})$

$=\hat{a}\cdot\hat{a}-\hat{a}\cdot\hat{b}-\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b}=1-\hat{a}\cdot\hat{b}-\hat{a}\cdot\hat{b}+1$

$=2-2\hat{a}\cdot\hat{b}=2-(-1)$ [Using (i)]

$=3$

$\therefore |\hat{a}-\hat{b}|=\sqrt{3}$

65. Given, $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$

Now, $\vec{a}+\vec{b}=4\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{a}-\vec{b}=-2\hat{i}+3\hat{j}-5\hat{k}$

Now, $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=(4\hat{i}+\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-5\hat{k})$

$=(4)(-2)+(1)(3)+(-1)(-5)=-8+3+5=0$

Hence, $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

66. Here $\vec{a}=4\hat{i}-\hat{j}+8\hat{k}$ and $\vec{b}=-\hat{j}+\hat{k}$

Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 8\\ 0 & -1 & 1\end{vmatrix}$$

$=\hat{i}(-1+8)-\hat{j}(4-0)+\hat{k}(-4-0)=7\hat{i}-4\hat{j}-4\hat{k}$

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

$=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\pm\frac{7\hat{i}-4\hat{j}-4\hat{k}}{\sqrt{49+16+16}}=\pm\frac{1}{9}(7\hat{i}-4\hat{j}-4\hat{k})$

67. We have, $|\vec{a}|=2, |\vec{b}|=7$

$\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$

$\therefore \; |\vec{a}\times\vec{b}|=|3\hat{i}+2\hat{j}+6\hat{k}|=\sqrt{3^2+2^2+6^2}$

$\therefore |\vec{a}\times\vec{b}|=\sqrt{49}=7$

Let 'θ' be the angle between $\vec{a}$ and $\vec{b}$, then we have

$\therefore \sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{7}{2\times7}=\frac{1}{2}$

$\sin\theta=\frac{1}{2} \quad \therefore \theta=\frac{\pi}{6}$

68. Let θ be the angle between $\vec{a}$ and $\vec{b}$.

We have, $(\vec{a}\times\vec{b})^2=|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2\cdot\sin^2\theta=|\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)$

$(\because |\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta)$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{a}|^2|\vec{b}|^2\cos^2\theta=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2$

Hence, $(\vec{a}\times\vec{b})^2=\vec{a}^2\vec{b}^2-(\vec{a}\cdot\vec{b})^2$

69. Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

Now, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow\ (\hat{i}-2\hat{j}+3\hat{k})\cdot(3\hat{i}-2\hat{j}+\hat{k})=\sqrt{(1)^2+(-2)^2+(3)^2}\times\sqrt{(3)^2+(-2)^2+(1)^2}\cos\theta$

$\Rightarrow\ 3+4+3=\sqrt{14}\times\sqrt{14}\cos\theta \Rightarrow \cos\theta=\frac{10}{14}$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{100}{196}}=\sqrt{\frac{96}{196}}$

$\Rightarrow\ \sin\theta=\frac{4\sqrt{6}}{14}=\frac{2\sqrt{6}}{7}$

70. We have, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

$\therefore\ \vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{a}-\vec{b}=0\hat{i}-\hat{j}-2\hat{k}$

A vector which is perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is given by

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}$$

$=-2\hat{i}+4\hat{j}-2\hat{k}=\vec{c}$ (say)

Now, $\vec{c}=\sqrt{(-2)^2+(4)^2+(-2)^2}=\sqrt{24}=2\sqrt{6}$

Required unit vector, $\hat{c}=\frac{\vec{c}}{|\vec{c}|}$

$=\frac{1}{2\sqrt{6}}(-2\hat{i}+4\hat{j}-2\hat{k})=-\frac{\hat{i}}{\sqrt{6}}+\frac{2\hat{j}}{\sqrt{6}}-\frac{\hat{k}}{\sqrt{6}}$

71. We have, $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow\ \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$

$\Rightarrow\ |\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ ... (i)

Similarly, $\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}=0$

$\Rightarrow\ \vec{a}\cdot\vec{b}+|\vec{b}|^2+\vec{b}\cdot\vec{c}=0$ ... (ii)

And, $\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}\cdot\vec{c}=0$

$\Rightarrow\ \vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+|\vec{c}|^2=0$ ... (iii)

Adding (i), (ii) and (iii), we get

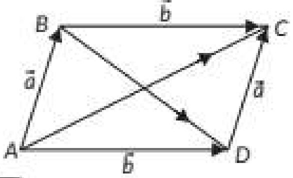
$|\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{a}\cdot\vec{b}+|\vec{b}|^2+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+|\vec{c}|^2=0$

$\Rightarrow\ |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c})=0$

$\Rightarrow\ (3)^2+(4)^2+(2)^2+2\mu=0$

$\Rightarrow\ \mu=\frac{-(9+16+4)}{2}=\frac{-29}{2}$

72. Let $\vec{a}=2\hat{i}-4\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+3\hat{k}$



Then diagonal $\overline{AC}$ of the parallelogram is $\vec{p}=\vec{a}+\vec{b}$

$\Rightarrow\ \vec{p}=2\hat{i}-4\hat{j}-5\hat{k}+2\hat{i}+2\hat{j}+3\hat{k}=4\hat{i}-2\hat{j}-2\hat{k}$

Therefore, unit vector parallel to it is

$\frac{\vec{p}}{|\vec{p}|}=\frac{4\hat{i}-2\hat{j}-2\hat{k}}{\sqrt{16+4+4}}=\frac{2\hat{i}-\hat{j}-\hat{k}}{\sqrt{6}}$

Now, diagonal $\overline{BD}$ of the parallelogram is

$\vec{p}'=\vec{b}-\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}-2\hat{i}+4\hat{j}+5\hat{k}=6\hat{j}+8\hat{k}$

Therefore, unit vector parallel to it is

$\frac{\vec{p}'}{|\vec{p}'|}=\frac{6\hat{j}+8\hat{k}}{\sqrt{36+64}}=\frac{6\hat{j}+8\hat{k}}{10}=\frac{3\hat{j}+4\hat{k}}{5}$

Now, $\vec{p}\times\vec{p}'=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & -2\\ 0 & 6 & 8\end{vmatrix}$

$=\hat{i}(-16+12)-\hat{j}(32-0)+\hat{k}(24-0)$

$=-4\hat{i}-32\hat{j}+24\hat{k}$

$\therefore$ Area of parallelogram $=\frac{|\vec{p}\times\vec{p}'|}{2}$

$=\frac{\sqrt{16+1024+576}}{2}=2\sqrt{101}$ sq. units.

Concept Applied

- Area of parallelogram $=\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$ where $\vec{d}_1$ and $\vec{d}_2$ diagonals of parallelograms.

73. Given, $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular

$\therefore\ \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ ...(i)

Also, $|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let α be the angle between $(2\vec{a}+\vec{b}+2\vec{c})$ and $\vec{a}$

$\therefore\ \cos\alpha=\frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$

$\Rightarrow\ \cos\alpha=\frac{2\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+2\vec{c}\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$ [From (i)]

$\cos\alpha=\frac{2|\vec{a}|^2}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}=\frac{2|\vec{a}|}{|2\vec{a}+\vec{b}+2\vec{c}|}$ ...(ii)

Let β be the angle between $(2\vec{a}+\vec{b}+2\vec{c})$ and $\vec{c}$

$\therefore\ \cos\beta=\frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|}$

$\Rightarrow\ \cos\beta=\frac{2\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|}$

$\Rightarrow\ \cos\beta=\frac{2|\vec{c}|^2}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|}$ [From (i)]

$\Rightarrow\ \cos\beta=\frac{2|\vec{c}|}{|2\vec{a}+\vec{b}+2\vec{c}|}$ ...(iii)

As $|\vec{a}|=|\vec{c}|$

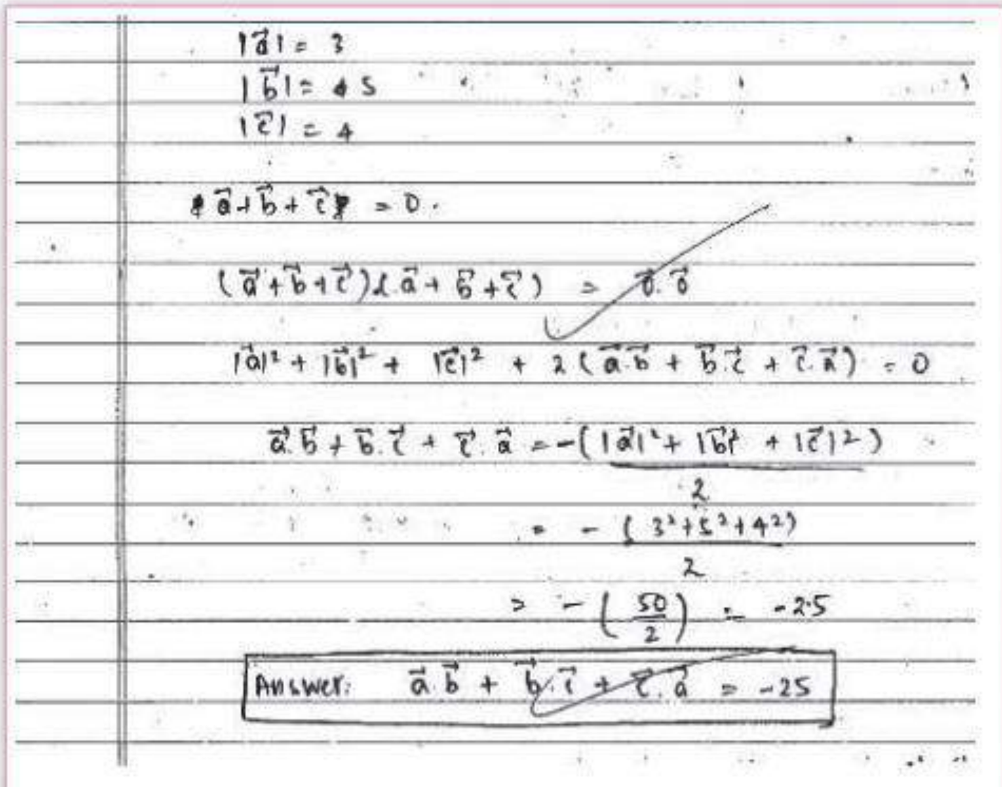
From (ii) & (iii), $\cos\alpha=\cos\beta \Rightarrow \alpha=\beta$

Hence, $(2\vec{a}+\vec{b}+2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$.

Angle between $\vec{a}$ and $(2\vec{a}+\vec{b}+2\vec{c})$ is

$\alpha=\cos^{-1}\left(\frac{2|\vec{a}|}{|2\vec{a}+\vec{b}+2\vec{c}|}\right)$

74.


$|\vec{a}| = 3$

$|\vec{b}| = 5$

$|\vec{c}| = 4$

$\vec{a} + \vec{b} + \vec{c} = 0.$

$(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = \vec{0}.\vec{0}$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)}{2}$

$= -\frac{(3^2+5^2+4^2)}{2}$

$= -\left(\frac{50}{2}\right) = -25$

Answer: $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -25$

[Topper's Answer, 2022]

75. We have, $|\vec{a}|=|\vec{b}|$

To prove, $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}=\cot\frac{\alpha}{2}$

i.e., $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|\cot\frac{\alpha}{2}$

i.e., $|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2\cot^2\frac{\alpha}{2}$

L.H.S. $=|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}$

$=2|\vec{a}|^2+2|\vec{a}||\vec{b}|\cos\alpha$ $(\because |\vec{a}|=|\vec{b}|)$ $=2|\vec{a}|^2+2|\vec{a}|^2\cos\alpha$

$=2|\vec{a}|^2(1+\cos\alpha)=2|\vec{a}|^2\,2\cos^2\frac{\alpha}{2}$

$=4|\vec{a}|^2\cos^2\frac{\alpha}{2}$ ... (i)

R.H.S. $=|\vec{a}-\vec{b}|^2\cot^2\frac{\alpha}{2}=\left(|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}\right)\cot^2\left(\frac{\alpha}{2}\right)$

$=(2|\vec{a}|^2-2|\vec{a}||\vec{b}|\cos\alpha)\cdot\cot^2\left(\frac{\alpha}{2}\right)$

$=2|\vec{a}|^2(1-\cos\alpha)\cot^2\frac{\alpha}{2}=2|\vec{a}|^2\cdot 2\sin^2\frac{\alpha}{2}\cdot\frac{\cos^2\alpha/2}{\sin^2\alpha/2}$

$=4|\vec{a}|^2\cos^2\alpha/2$ ...(ii)

$\therefore$ From (i) and (ii)
L.H.S. = R.H.S.

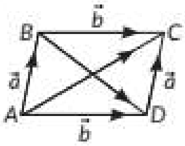
76. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$

Then diagonal $\overline{AC}$ of the parallelogram is

$\vec{p}=\vec{a}+\vec{b}$

$=\hat{i}+2\hat{j}+3\hat{k}+2\hat{i}+4\hat{j}-5\hat{k}$

$=3\hat{i}+6\hat{j}-2\hat{k}$



Therefore unit vector parallel to it is

$\frac{\vec{p}}{|\vec{p}|}=\frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}}=\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$

Now, diagonal $\overline{BD}$ of the parallelogram is

$\vec{p}'=\vec{b}-\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}=\hat{i}+2\hat{j}-8\hat{k}$

Therefore unit vector parallel to it is

$\frac{\vec{p}'}{|\vec{p}'|}=\frac{\hat{i}+2\hat{j}-8\hat{k}}{\sqrt{1+4+64}}=\frac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$

Concept Applied

- In a parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$, one of the diagonal is $\vec{a}+\vec{b}$ and the other is $\vec{b}-\vec{a}$.

77. Given, ΔABC with vertices

$A(1,2,3)=\hat{i}+2\hat{j}+3\hat{k}$, $B(2,-1,4)=2\hat{i}-\hat{j}+4\hat{k}$,

$C(4,5,-1)=4\hat{i}+5\hat{j}-\hat{k}$

Now $\overline{AB}=\overline{OB}-\overline{OA}=(2\hat{i}-\hat{j}+4\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=\hat{i}-3\hat{j}+\hat{k}$.

$\overline{AC}=\overline{OC}-\overline{OA}=(4\hat{i}+5\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=3\hat{i}+3\hat{j}-4\hat{k}$.

$\therefore\ (\overline{AB}\times\overline{AC})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-3&1\\3&3&-4\end{vmatrix}=9\hat{i}+7\hat{j}+12\hat{k}$

Hence, area of $\Delta ABC=\frac{1}{2}|\overline{AB}\times\overline{AC}|=\frac{1}{2}|9\hat{i}+7\hat{j}+12\hat{k}|$

$=\frac{1}{2}\sqrt{9^2+7^2+12^2}=\frac{1}{2}\sqrt{81+49+144}=\frac{1}{2}\sqrt{274}$ sq. units

78. Since $\vec{a},\vec{b}$ and $\vec{c}$ are the position vector of A, B and C respectively.

Then $\overline{BC}$ = position vector of C – position vector of B

$=\vec{c}-\vec{b}$ ...(i)

and $\overline{CA}$ = position vector of A – position vector of C

$=\vec{a}-\vec{c}$ ...(ii)

A, B and C are collinear if and only if $\overline{BC}\times\overline{CA}=\vec{0}$

if and only if $(\vec{c}-\vec{b})\times(\vec{a}-\vec{c})=\vec{0}$ {From (i) and (ii)}

if and only if $(\vec{c}\times\vec{a})-(\vec{c}\times\vec{c})-(\vec{b}\times\vec{a})+(\vec{b}\times\vec{c})=\vec{0}$

if and only if $(\vec{c}\times\vec{a})+(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})=\vec{0}$

$\{\because \vec{c}\times\vec{c}=\vec{0} \text{ and } \vec{a}\times\vec{b}=-\vec{b}\times\vec{a}\}$

iff $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\vec{0}$

$\therefore$ A, B and C are collinear iff $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\vec{0}$

79. Here, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$

$\Rightarrow \vec{b}+\vec{c}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

The unit vector along $\vec{b}+\vec{c}$ is $\vec{p}=\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}$

$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$

Also, $\vec{a}.\vec{p}=1$ (Given)

$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1 \Rightarrow \sqrt{\lambda^2+4\lambda+44}=\lambda+6$

$\Rightarrow \lambda^2+4\lambda+44=\lambda^2+12\lambda+36 \Rightarrow 8\lambda=8 \Rightarrow \lambda=1$

$\therefore$ The required unit vector

$\vec{p}=\frac{(2+1)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{1+4+44}}=\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k}).$

80. Given, position vector of $A=\hat{i}+\hat{j}+\hat{k}$

Position vector of $B=2\hat{i}+5\hat{j}$

Position vector of $C=3\hat{i}+2\hat{j}-3\hat{k}$

Position vector of $D=\hat{i}-6\hat{j}-\hat{k}$

$\therefore \overline{AB}=(2\hat{i}+5\hat{j})-(\hat{i}+\hat{j}+\hat{k})=\hat{i}+4\hat{j}-\hat{k}$

and $\overline{CD}=(\hat{i}-6\hat{j}-\hat{k})-(3\hat{i}+2\hat{j}-3\hat{k})=-2\hat{i}-8\hat{j}+2\hat{k}$

Now, $|\overline{AB}|=\sqrt{(1)^2+(4)^2+(-1)^2}=\sqrt{18}$

$|\overline{CD}|=\sqrt{(-2)^2+(-8)^2+(2)^2}=\sqrt{4+64+4}=\sqrt{72}=2\sqrt{18}$

$=\sqrt{72}=2\sqrt{18}$

Let θ be the angle between $\overline{AB}$ and $\overline{CD}$.

$\therefore \cos\theta=\frac{\overline{AB}\cdot\overline{CD}}{|\overline{AB}||\overline{CD}|}=\frac{(\hat{i}+4\hat{j}-\hat{k})\cdot(-2\hat{i}-8\hat{j}+2\hat{k})}{(\sqrt{18})(2\sqrt{18})}$

$=\frac{-2-32-2}{36}=\frac{-36}{36}=-1 \Rightarrow \cos\theta=-1 \Rightarrow \theta=\pi$

Since, angle between $\overline{AB}$ and $\overline{CD}$ is 180°.

$\therefore$ $\overline{AB}$ and $\overline{CD}$ are collinear.

81. We have, $|\vec{a}|=1, |\vec{b}|=2$ and $|\vec{c}|=3$ ...(i)

Given, Projection of $\vec{b}$ along $\vec{a}$ = Projection of $\vec{c}$ along $\vec{a}$

$\Rightarrow \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} \Rightarrow \vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$

$\Rightarrow \vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ ...(ii)

Given, $\vec{b}$ and $\vec{c}$ are perpendicular to each other

$\therefore \vec{b}\cdot\vec{c}=0$ ...(iii)

Now, $|3\vec{a}-2\vec{b}+2\vec{c}|^2=(3\vec{a}-2\vec{b}+2\vec{c})\cdot(3\vec{a}-2\vec{b}+2\vec{c})$

$=9(\vec{a}\cdot\vec{a})-6(\vec{a}\cdot\vec{b})+6(\vec{a}\cdot\vec{c})-6(\vec{b}\cdot\vec{a})+4(\vec{b}\cdot\vec{b})-4(\vec{b}\cdot\vec{c})$
$+6(\vec{c}\cdot\vec{a})-4(\vec{c}\cdot\vec{b})+4(\vec{c}\cdot\vec{c})$

$=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2+2\{-6(\vec{a}\cdot\vec{b})-4(\vec{b}\cdot\vec{c})+6(\vec{a}\cdot\vec{c})\}$

$=9\times1^2+4\times2^2+4\times3^2+2\{-6(\vec{a}\cdot\vec{b})-4\times0+6(\vec{a}\cdot\vec{b})\}$

[From eqn (i) and (iii)]

$=9+16+36+2\times0=61$ [From eqn. (ii)]

$\therefore |3\vec{a}-2\vec{b}+2\vec{c}|=\sqrt{61}$

82. Let $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}$

Now, it is given that, $\vec{d}$ is perpendicular to

$\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$ $\therefore \vec{d}\cdot\vec{b}=0$ and $\vec{d}\cdot\vec{c}=0$

$\Rightarrow x-4y+5z=0$ ...(i)

and $3x+y-z=0$ ...(ii)

Also, $\vec{d}\cdot\vec{a}=21$, where $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$

$\Rightarrow 4x+5y-z=21$ ...(iii)

Eliminating z from (i) and (ii), we get $16x+y=0$ ...(iv)

Eliminating z from (ii) and (iii), we get $x+4y=21$ ...(v)

Solving (iv) and (v), we get $x=\frac{-1}{3}, y=\frac{16}{3}$

Putting the values of x and y in (i), we get $z=\frac{13}{3}$

$\therefore \vec{d}=\frac{-1}{3}\hat{i}+\frac{16}{3}\hat{j}+\frac{13}{3}\hat{k}$ is the required vector.

Concept Applied

➲ If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $a\cdot b=a_1b_1+a_2b_2+a_3b_3$.

83. Given, $|\vec{a}|=|\vec{b}|=|\vec{c}|$ ...(i)

and $\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$ ...(ii)

Let $(\vec{a}+\vec{b}+\vec{c})$ be inclined to vectors $\vec{a}, \vec{b}$ and $\vec{c}$ by angles α, β and γ respectively. Then

$\cos\alpha=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$

$=\frac{|\vec{a}|^2+0+0}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$ [Using (ii)]

$=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ...(iii)

Similarly, $\cos\beta=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ...(iv)

and $\cos\gamma=\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$ ...(v)

From (i), (iii), (iv) and (v), we get

$\cos\alpha=\cos\beta=\cos\gamma \Rightarrow \alpha=\beta=\gamma$

Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to the vector $\vec{a}, \vec{b}$ and $\vec{c}$.

Also, the angle between them is given as

$\alpha=\cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right), \beta=\cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$

$\gamma=\cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$

84. We have, $A(2\hat{i}-\hat{j}+\hat{k}), B(\hat{i}-3\hat{j}-5\hat{k})$,

and $C(3\hat{i}-4\hat{j}-4\hat{k})$

Then, $\overline{AB}=(1-2)\hat{i}+(-3+1)\hat{j}+(-5-1)\hat{k}=-\hat{i}-2\hat{j}-6\hat{k}$,

$\overline{AC}=(3-2)\hat{i}+(-4+1)\hat{j}+(-4-1)\hat{k}=\hat{i}-3\hat{j}-5\hat{k}$

and $\overline{BC}=(3-1)\hat{i}+(-4+3)\hat{j}+(-4+5)\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

Now angle between $\overline{AC}$ and $\overline{BC}$ is given by

$\cos\theta=\frac{(\overline{AC})\cdot(\overline{BC})}{|\overline{AC}||\overline{BC}|}=\frac{2+3-5}{\sqrt{1+9+25}.\sqrt{4+1+1}}$

$\Rightarrow \cos\theta=0 \Rightarrow \theta=\frac{\pi}{2} \Rightarrow BC\perp AC$

So, A, B, C are vertices of right angled triangle.

Now area of $\Delta ABC=\frac{1}{2}|\overline{AC}\times\overline{BC}|$

$=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & -5\\ 2 & -1 & 1\end{matrix}\right\|=\frac{1}{2}|(-3-5)\hat{i}-(1+10)\hat{j}+(-1+6)\hat{k}|$

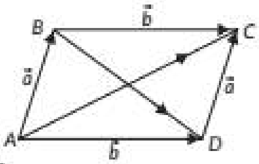
$=\frac{1}{2}|-8\hat{i}-11\hat{j}+5\hat{k}|=\frac{1}{2}\sqrt{64+121+25}=\frac{\sqrt{210}}{2}$ sq.units.

Concept Applied

- If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

85. Let $\vec{a}=2\hat{i}-4\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+3\hat{k}$



Then diagonal $\overline{AC}$ of the parallelogram is $\vec{p}=\vec{a}+\vec{b}$

$\Rightarrow \vec{p}=2\hat{i}-4\hat{j}-5\hat{k}+2\hat{i}+2\hat{j}+3\hat{k}=4\hat{i}-2$

Therefore, unit vector parallel to it is

$\frac{\vec{p}}{|\vec{p}|}=\frac{4\hat{i}-2\hat{j}-2\hat{k}}{\sqrt{16+4+4}}=\frac{2\hat{i}-\hat{j}-\hat{k}}{\sqrt{6}}$

Now, diagonal $\overline{BD}$ of the parallelogram is

$\vec{p}'=\vec{b}-\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}-2\hat{i}+4\hat{j}+5\hat{k}=6\hat{j}+8\hat{k}$

Therefore, unit vector parallel to it is

$\frac{\vec{p}'}{|\vec{p}'|}=\frac{6\hat{j}+8\hat{k}}{\sqrt{36+64}}=\frac{6\hat{j}+8\hat{k}}{10}=\frac{3\hat{j}+4\hat{k}}{5}$

Now, $\vec{p}\times\vec{p}'=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & -2\\ 0 & 6 & 8\end{vmatrix}$

$=\hat{i}(-16+12)-\hat{j}(32-0)+\hat{k}(24-0)=-4\hat{i}-32\hat{j}+24\hat{k}$

$\therefore$ Area of parallelogram $=\frac{|\vec{p}\times\vec{p}'|}{2}$

$=\frac{\sqrt{16+1024+576}}{2}=2\sqrt{101}$ sq. units.

Concept Applied

- Area of parallelogram $=\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$ where $\vec{d}_1$ and $\vec{d}_2$ diagonals of parallelograms.

86. Two non zero vectors are parallel if and only if their cross product is zero vector.

So, we have to prove that cross product of $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ is zero vector.

$(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=(\vec{a}\times\vec{b})-(\vec{a}\times\vec{c})-(\vec{d}\times\vec{b})+(\vec{d}\times\vec{c})$

Since, it is given that $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$.

And, $\vec{d}\times\vec{b}=-\vec{b}\times\vec{d}$, $\vec{d}\times\vec{c}=-\vec{c}\times\vec{d}$

Therefore, $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=(\vec{c}\times\vec{d})-(\vec{b}\times\vec{d})+(\vec{b}\times\vec{d})-(\vec{c}\times\vec{d})=\vec{0}$

Hence, $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

87. $(\vec{r}\times\hat{i})\cdot(\vec{r}\times\hat{j})+xy=[(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}]\cdot[(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{j})]+xy$

$=(-y\hat{k}+z\hat{j})\cdot(x\hat{k}-z\hat{i})+xy=-xy+xy=0$

88. Here, $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{i}-4\hat{j}-5\hat{k}$

$\therefore \vec{a}-\vec{b}=(\hat{i}+2\hat{j}+\hat{k})-(2\hat{i}+\hat{j})=-\hat{i}+\hat{j}+\hat{k}$

and $\vec{c}-\vec{b}=(3\hat{i}-4\hat{j}-5\hat{k})-(2\hat{i}+\hat{j})=\hat{i}-5\hat{j}-5\hat{k}$

Vector perpendicular to both $\vec{a}-\vec{b}$ and $\vec{c}-\vec{b}$ is

$(\vec{a}-\vec{b})\times(\vec{c}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & 1\\ 1 & -5 & -5\end{vmatrix}$

$=(-5+5)\hat{i}-(5-1)\hat{j}+(5-1)\hat{k}=-4\hat{j}+4\hat{k}$

$\therefore$ Unit vector perpendicular to both $\vec{a}-\vec{b}$ and $\vec{c}-\vec{b}$

$=\pm\frac{-4\hat{j}+4\hat{k}}{|-4\hat{j}+4\hat{k}|}=\pm\frac{-4\hat{j}+4\hat{k}}{\sqrt{(-4)^2+4^2}}=\pm\frac{-4\hat{j}+4\hat{k}}{4\sqrt{2}}=\pm\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$.

89. Given $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$

We have $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c} \Rightarrow |\vec{a}+\vec{b}|^2=|-\vec{c}|^2 \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2(\vec{a}\cdot\vec{b})=|\vec{c}|^2$

$\Rightarrow 9+25+2|\vec{a}||\vec{b}|\cos\theta=49 \Rightarrow 2\times3\times5\times\cos\theta=49-34=15$

$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}=60°$

90. We have, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Let $\vec{r}=\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{p}=\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

A unit vector perpendicular to both $\vec{r}$ and $\vec{p}$ is given as $\pm\frac{\vec{r}\times\vec{p}}{|\vec{r}\times\vec{p}|}$.

Now, $\vec{r}\times\vec{p}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=-2\hat{i}+4\hat{j}-2\hat{k}$

So, the required unit vector is

$=\pm\frac{(-2\hat{i}+4\hat{j}-2\hat{k})}{\sqrt{(-2)^2+4^2+(-2)^2}}=\mp\frac{(\hat{i}-2\hat{j}+\hat{k})}{\sqrt{6}}$.

91. Here, $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$ and $\vec{c} = 2\hat{j} - \hat{k}$

$\therefore \quad \vec{a} + \vec{b} = (2\hat{i} - 3\hat{j} + \hat{k}) + (-\hat{i} + \hat{k}) = \hat{i} - 3\hat{j} + 2\hat{k}$

and $\vec{b} + \vec{c} = (-\hat{i} + \hat{k}) + (2\hat{j} - \hat{k}) = -\hat{i} + 2\hat{j}$

$\therefore (\vec{a} + \vec{b}) \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{i} - 2\hat{j} - \hat{k}$

$\therefore$ Area of a parallelogram whose diagonals are $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$

$= \frac{1}{2}\left|(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})\right| = \frac{1}{2}\left|-4\hat{i} - 2\hat{j} - \hat{k}\right|$

$= \frac{1}{2}\sqrt{(-4)^2 + (-2)^2 + (-1)^2} = \frac{\sqrt{21}}{2}$ sq.units.

92. Let $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

Now, it is given that $\vec{p}$ is perpendicular to both $\vec{\alpha}$ and $\vec{\beta}$

$\therefore \quad \vec{p} \cdot \vec{\alpha} = 0$ and $\vec{p} \cdot \vec{\beta} = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (4\hat{i} + 5\hat{j} - \hat{k}) = 0$

and $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - 4\hat{j} + 5\hat{k}) = 0$

$\Rightarrow \quad 4x + 5y - z = 0$ ...(i)

and $x - 4y + 5z = 0$ ...(ii)

Also, we have, $\vec{p} \cdot \vec{q} = 21 \Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) = 21$

$\Rightarrow \quad 3x + y - z = 21$ ...(iii)

Eliminating z from (i) and (ii), we get

$21x + 21y = 0 \Rightarrow x + y = 0$ ...(iv)

Eliminating z from (i) and (iii), we get $x + 4y = -21$ ...(v)

Solving (iv) and (v), we get $x = 7, y = -7$

Now, from (i), we get $z = -7$

So, $\vec{p} = 7\hat{i} - 7\hat{j} - 7\hat{k}$.

CBSE Sample Questions

1. Let $\vec{a}$ be the unit vector in the direction opposite to the given vector $\left(-\frac{3}{4}\hat{j}\right)$.

Then, $\vec{a} = \frac{-1}{\sqrt{\left(\frac{3}{4}\right)^2}}\left(-\frac{3}{4}\hat{j}\right) = \hat{j}$ (1)

2. A vector in the direction opposite to $2\hat{i} + 3\hat{j} - 6\hat{k}$ is $-2\hat{i} - 3\hat{j} + 6\hat{k}$.

Its magnitude is $|\sqrt{4+9+36}| = |\sqrt{49}| = 7$

So, a vector in the direction opposite to $2\hat{i} + 3\hat{j} - 6\hat{k}$ of magnitude 5 units is $\frac{5}{7}(-2\hat{i} - 3\hat{j} + 6\hat{k})$ (1)

3. (a): Scalar projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

Scalar projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k}$

$= \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{|\hat{i} + 2\hat{j} - 3\hat{k}|} = \frac{7}{\sqrt{14}}$ (1)

4. (b): $|\vec{a} - 2\vec{b}|^2 = (\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b})$

$= \vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + 4\vec{b} \cdot \vec{b} = |\vec{a}|^2 - 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2 = 4 - 16 + 36 = 24$

$\therefore \quad |\vec{a} - 2\vec{b}| = 2\sqrt{6}$ (1)

5. Area of the triangle

$= \frac{1}{2}|2\hat{i} \times (-3)\hat{j}| = \frac{1}{2}|-6\hat{k}| = 3$ sq. units (1)

6. We have, $|\hat{a} + \hat{b}|^2 = 1 \Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a} \cdot \hat{b} = 1$

$\Rightarrow \quad 2\hat{a} \cdot \hat{b} = 1 - 1 - 1$ $\quad (\because |\hat{a}| = |\hat{b}| = 1)$

$\Rightarrow \quad \hat{a} \cdot \hat{b} = \frac{-1}{2} \Rightarrow |\hat{a}||\hat{b}|\cos\theta = \frac{-1}{2} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \quad \theta = \pi - \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$ (1)

7. Since $\vec{a}$ is a unit vector, $\therefore |\vec{a}| = 1$

Now, $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

$\Rightarrow \quad \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$ (1/2)

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12 \quad (\because \vec{a} \cdot \vec{x} = \vec{x} \cdot \vec{a})$ (1/2)

$\Rightarrow \quad |\vec{x}|^2 - 1 = 12 \Rightarrow |\vec{x}|^2 = 13 \Rightarrow |\vec{x}| = \sqrt{13}$ (1)

8. Since, $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b})$ (1)

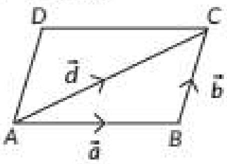
$\therefore \quad |\vec{a} + \vec{b}|^2 = 1 + 1 + 2\cos\theta$ [As $|\vec{a}| = |\vec{b}| = 1$]

$= 2(1 + \cos\theta) = 4\cos^2\frac{\theta}{2}$ $\quad \left[\because 1 + \cos\theta = 2\cos^2\frac{\theta}{2}\right]$ (1/2)

$\therefore \quad |\vec{a} + \vec{b}| = 2\cos\frac{\theta}{2}$ (1/2)

9. Let *ABCD* is a parallelogram such that

$\vec{a} = \overline{AB} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \overline{BC}$, and $\vec{d} = \overline{AC} = 4\hat{i} + 5\hat{k}$.



Now, $\vec{a} + \vec{b} = \vec{d}$ (By triangle law)

$\Rightarrow \vec{b} = \vec{d} - \vec{a}$

$\Rightarrow \quad \vec{b} = (4\hat{i} + 5\hat{k}) - (\hat{i} - \hat{j} + \hat{k})$

$= 3\hat{i} + \hat{j} + 4\hat{k}$ (1/2)

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -5\hat{i} - \hat{j} + 4\hat{k}$ (1)

$\therefore$ Area of parallelogram $= |\vec{a} \times \vec{b}|$

$= \sqrt{25 + 1 + 16} = \sqrt{42}$ sq.units (1/2)

10. We have, $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \quad (\vec{b} - \vec{c}) = \vec{0}$ or $\vec{a} \perp (\vec{b} - \vec{c})$

$\Rightarrow \quad \vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b} - \vec{c})$ (1)

Also, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$

$\Rightarrow \quad \vec{b} - \vec{c} = \vec{0}$ or $\vec{a} \parallel (\vec{b} - \vec{c})$

$\Rightarrow \quad \vec{b} = \vec{c}$ or $\vec{a} \parallel (\vec{b} - \vec{c})$ (1)

Since, $\vec{a}$ can not be both perpendicular to $(\vec{b} - \vec{c})$ and parallel to $(\vec{b} - \vec{c})$.

Hence, $\vec{b} = \vec{c}$. (1)